Evaluation of neutrino masses from $m_{\beta\beta}$ values

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A neutrino mass matrix is considered under conditions of a CP invariance and a small reactor mixing θ_{13} angle. Absolute mass values for three neutrinos are evaluated for normal and inverted hierarchy spectra on the ground of data for oscillation mixing neutrino parameters and an effective neutrino mass $m_{\beta\beta}$ related to a probability of neutrinoless two beta decay.

1. Introduction

Discovering of neutrino oscillations, which have been predicted in the Refs. [1, 2], points to a nonconservation of lepton quantum numbers L_e, L_μ, L_τ and nonequal each other neutrino masses. Now further objectives for neutrino physics investigations are a clarification of Dirac or Majorana nature for neutrinos, a refinement of neutrino mixing parameters and a determination of neutrino mass absolute values [3, 4].

Different theoretical approaches may be used to solve these problems, for instance, Grand Unification Theories (GUTs) or various phenomenological models (PMs). The characteristical feature for many neutrino physics articles is the use of continuous or discrete symmmetries beyond the known symmmetries of the Standard Model (SM). For instance, absolute mass values of three neutrino were obtained in the framework of the three-flavor model for Majorana neutrinos with the Pauli symmetry ($0,054\,eV < m_{1,2} < 0,391\,eV$, $0,058\,eV < m_3 < 0,406\,eV$) [5]. In the PM context interesting results were also obtained: the phenomenological scheme of three-bimaximal mixing of neutrinos, where the explicit form (see Sec.4) of neutrino mixing matrix was given in Ref.[6]; the relations between quark and neutrino mixing angles [7], which, e.g., were used in Ref. [8] for precise neutrino mixing angles evaluation; constraints on the neutrino mass matrix structure and dependences between the minimal neutrino mass and the effective neutrino mass related to a probability of neutrinoless two beta decay $m_{\beta\beta}$, these dependences were presented in the graphical form in Refs.[9, 10]. There are many papers [11, 12, 13, 14] about a neutrino mass matrix structure, in particular about a number and locations of zeroth matrix elements.

The present paper is devoted to an examination of a neutrino mass matrix structure and an elucidation a role of some assumptions, which make possible to find absolute neutrino mass values on the basis of the minimally extended SM with three types of massive neutrinos [3, 4]. The set of existing experimental data concerning neutrino oscillations [20] has a basic role in obtaining of neutrino mass values. In Sec.2 a general neutrino mass matrix structure for Majorana neutrinos in the flavor representation is presented together with its link to the neutrino mixing matrix under the CP invariance condition. In Sec.

3 additional simple assumptions are considered, which facilitate neutrino mass matrix structure and allow to evaluate absolute neutrino masses at specified $m_{\beta\beta}$ values. In doing so neutrino masses became dependent on mixing angles. It should be noted that the main assumption, that used throughout the present paper, is the condition of the small reactor mixing θ_{13}^{ν} angle, that is in accordance with the experimental data (e.g., see [20, 26]). In Sec.4 neutrino mass evaluations are performed at prescribed $m_{\beta\beta}$ values from an acceptable range and experimentally determined oscilation mixing parameters of neutrinos. As a check on performed calculations the limiting case is treated when $m_{\beta\beta}$ equals to zero and the three-bimaximal neutrino mixing takes place. In the last section the results obtained are discussed and their possible generalization for nonzero θ_{13}^{ν} values are considered.

2. Neutrino mixing matrix and neutrino mass matrix

As it is known, left components of fields of flavor $\nu_\alpha, \alpha = e, \mu, \tau$ and massive $\nu_i, i = 1, 2, 3$ neutrinos are connected with the Pontecorvo-Maki-Nakagava-Sakata matrix:

$$\nu_{\alpha L} = \sum_{i=1}^{3} U_{PMNS,\alpha i} \nu_{iL} \tag{1}$$

In general case when neutrinos are Majorana particles the U_{PMNS} matrix may be specified in the form: $U^D_{PMNS} \cdot P^M$, where matrix U^D_{PMNS} coincides with the mixing matrix of Dirac neutrinos, while P^M is the diagonal matrix containing additional phases due to CP noninvariance in processes involving Majorana neutrinos. Let us write the U_{PMNS} matrix in the standard parameterization apart from new notations for mixing angles and phases, which differ from angles and phases notations in Cabibbo-Kobayashi-Maskava matrix. The Dirac phase δ is denoted as ϵ, angles θ_{ij}^{ν} are denoted as η_{ij}, so $c_{ij} \equiv \cos\eta_{ij}$ $s_{ij} \equiv \sin\eta_{ij}$, while ϕ_1, ϕ_2 are the Majorana phases.

$$U_{PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\epsilon} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\epsilon} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\epsilon} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\epsilon} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\epsilon} & c_{13}c_{23} \end{pmatrix} \cdot \begin{pmatrix} e^{-i\phi_1} & 0 & 0 \\ 0 & e^{-i\phi_2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{2}$$

It is common knowledge that at present experimental data concerning phases ϵ, ϕ_1 and ϕ_2 are absent. Moreover, these phases will not be observable for a long period of time due to the small η_{13} value and difficulties in realization of experiments for the ϕ_1 and ϕ_2 determination. Thus we consider processes when the CP invariance holds and the reactor mixing angle η_{13} is negligible. In this case the neutrino mass matrix can be presented in the flavor basis in the form:

$$M_f^{\nu} = U_{PMNS} M_m^{\nu} U_{PMNS}^T, \tag{3}$$

where $M_m^{\nu} = diag\{m_1, m_2, m_3\}$. $m_i, i = 1, 2, 3$, can have minus and plus signs, and the U_{PMNS} matrix depends only on two mixing angles, namely η_{12} and η_{23}.

Note that the minus and plus m_i signs are associated with relative CP parities of neutrinos (e.g. see [3, 4]). Let us write the explicit form of the neutrino mass matrix taking into account the constraints, which have been made.

$$M_f^\nu = \begin{pmatrix} m_1c_{12}^2 + m_2s_{12}^2 & -m_1c_{12}s_{12}c_{23} + m_2c_{12}s_{12}c_{23} & m_1c_{12}s_{12}s_{23} - m_2c_{12}s_{12}s_{23} \\ -m_1s_{12}c_{23}c_{12} + m_2c_{12}c_{23}s_{12} & m_1s_{12}^2c_{23}^2 + m_2c_{12}^2c_{23}^2 + m_3s_{23}^2 & -m_1s_{12}^2c_{23}s_{23} - m_2c_{12}^2s_{23}c_{23} + m_3s_{23}c_{23} \\ m_1c_{12}s_{12}s_{23} - m_2c_{12}s_{12}s_{23} & -m_1s_{12}^2c_{23}s_{23} - m_2c_{12}^2c_{23}s_{23} + m_3c_{23}s_{23} & m_3c_{23}^2 + m_1s_{12}^2s_{23}^2 + m_2c_{12}^2s_{23}^2 \end{pmatrix} \quad (4)$$

It is well known that various assumptions are considered concerning matrix elements values in order to specify a particular structure of the M_f^ν matrix [11, 12]. For instance, conditions of zero values for a few matrix elements or the spur and the deternimant are used. However in the present paper we assume that the spur and the deternimant of the M_f^ν matrix are not equal to zero. Besides we use the condition, that a value of the first diagonal M_f^ν matrix element (or some restriction on this value) can be determinated in principle in experiments for a neutrinoless two beta decay search.

3. Additional restrictions on mixing angles and neutrino mass spectra.

As indicated above, in the neutrino PM framework additional assumptions on the mass matrix M_f^ν or its matrix elements are used in order to obtain some results to be tested in experiments. For example, in the paper [11] the case when the modulus of the diagonal matrix element M_{ee}^ν equals zero is investigated in detail. In Refs. [13, 14] cases have been considered when the spur SpM^ν or the determinant $DetM^\nu$

$$SpM^\nu = m_1 + m_2 + m_3, \quad DetM^\nu = m_1m_2m_3. \quad (5)$$

are equal to zero values. In the present paper we put that the spur and the determinat for the M_f^ν matrix are not equal to zero.

$$SpM^\nu \neq 0, \quad DetM^\nu \neq 0. \quad (6)$$

It is convenient to use the following classification for possible neutrino mass spectra: the case for $m_1 < m_2 \ll m_3$ will be called the neutrino mass spectrum with a normal hierarchy (NH), the case for $m_3 \ll m_1 < m_2$ will be called the neutrino mass spectrum with an inverted hierarchy (IH).

It is known a value of the first diagonal matrix element M_{ee}^ν is connected with a probability of a neutrinoless two beta decay of nucleus: $(A, Z) \to (A, Z+2) + 2e$. Actually the half-life of $0\nu2\beta$ decay $T_{1/2}^{0\nu2\beta}$ is in an inverse proportion to the square of a M_{ee}^ν modulus. For this reason the M_{ee}^ν modulus is usually denoted as $m_{\beta\beta}$ [15]. A search for a neutrinoless two beta decay is intended to reveal Dirac or Majorana nature of neutrinos. Moreover, a discovery of this decay makes possible to deternime an absolute scale of neutrino masses. The conditions for

picking of isotopes for a reliable detection of a signal of a neutrinoless two beta decay above the total background were considered in Ref.[16].

If one use the parameterization (2) provided CP invariance holds, then the matrix element M_{ee}^{ν} modulus can be written in the form:

$$m_{\beta\beta} \equiv |M_{ee}^{\nu}| = |m_1 c_{12}^2 c_{13}^2 + m_2 s_{12}^2 c_{13}^2 + m_3 s_{13}^2|, \tag{7}$$

however as $\eta_{13} \approx 0$, hence $m_{\beta\beta} \approx |m_1 c_{12}^2 + m_2 s_{12}^2|$.

Results of the experiments performed (Heidelberg-Moscow, IGEX) provide the upper limit for admissible $m_{\beta\beta}$ values (90% C.L.): $m_{\beta\beta} < 0,35 \div 1,05 eV$, $m_{\beta\beta} < 0,33 \div 1,35 eV$, respectively [17, 18, 19]. On the other hand there are data for oscillation neutrino mixing parameters, which were obtained in many experiments [20], such as the differencies of squares of neutrino masses Δm_{ij}^2 and mixing angles θ_{ij}:

$$\begin{gathered}
\Delta m_{21}^2 = (7,7 \div 8,3) \times 10^{-5} eV^2, \\
|\Delta m_{32}^2| = (1,9 \div 3,0) \times 10^{-3} eV^2, \\
\sin^2(2\theta_{12}) = 0.86(+0.03/-0.04), \\
\sin^2(2\theta_{23}) > 0.92, \quad \sin^2(2\theta_{13}) < 0.19
\end{gathered} \tag{8}$$

4. Evaluation of absolute values for neutrino masses

It is possible to evaluate absolute values for neutrino masses using at $\eta_{13} = 0$ the relation (7), experimental restrictions (8) as well as fixed ranges for $m_{\beta\beta}$ values. In the Table 1 neutrino mass values are presented in the five ranges for $0 \leq m_{\beta\beta} \leq 0,9 eV$, where m_3^1, m_3^2 denote the m_3 masses for the IH spectrum and the NH spectrum, correspondingly. Performing m_i, $i = 1,2,3$, masses evaluations the ranges of acceptable $m_{\beta\beta}$ values have been setted, in so doing the possibility of different m_i signs have been taken into account.

As a check on performed calculations one can use the limiting case when the matrix element M_{ee}^{ν} equals to zero and the three-bimaximal neutrino mixing takes place [1, 2]. As is known, experimental data for mixing angles are approximated well with the help of the three-bimaximal matrix:

$$U_{PMNS}^{TBM} = \begin{pmatrix} 2/\sqrt{6} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \\ 1/\sqrt{6} & -1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix} \tag{9}$$

If one use the condition $m_{\beta\beta} \approx 0$ as an admissible approximation together with the U_{PMNS}^{TBM} matrix (9), then the following neutrino mass values can be obtained:

$$\begin{gathered}
m_1 \approx (5,1 \div 5,3) \times 10^{-3} eV, \\
m_2 \approx (10,1 \div 10,5) \times 10^{-3} eV, \\
m_3 \approx (44,7 \div 55,8) \times 10^{-3} eV
\end{gathered} \tag{10}$$

Table 1. Neutrino masses evaluated against permissible $m_{\beta\beta}$ values

$m_{\beta\beta}$, eV	m_1, eV	m_2, eV	m_3^1, eV	m_3^2, eV
$(5 \pm 4) \cdot 10^{-1}$	$0.1 \div 0.9$	$0.1004 \div 0.9001$	$0.084 \div 0.899$	$0.11 \div 0.902$
$(5 \pm 4) \cdot 10^{-2}$	$0.009 \div 0.497$	$0.0124 \div 0.4971$	$0. \div 0.495$	$0.0453 \div 0.5$
$(5 \pm 4) \cdot 10^{-3}$	$0. \div 0.029$	$0.009 \div 0.03$	–	$0.044 \div 0.062$
$(5 \pm 4) \cdot 10^{-4}$	$0.004 \div 0.007$	$0.0096 \div 0.012$	–	$0.045 \div 0.056$
$(5 \pm 4) \cdot 10^{-5}$	$0.004 \div 0.005$	$0.0096 \div 0.011$	–	$0.045 \div 0.056$
≈ 0	$0.004 \div 0.005$	$0.0096 \div 0.011$	–	$0.045 \div 0.056$

It should be noted, that in the present paper the effective neutrino mass $m_{\beta\beta}$ for a neutrinoless two beta decay is used as the independent parameter in order to find m_i, $i = 1, 2, 3$ absolute values, while an inverse dependence is treated in Refs. [9, 10], that was displayed in the graphical form. The correspondence between the results of the Refs. [9, 10] and the present paper exists within the accuracy dependent on the accuracy of graphical representation and employed experimental data. Furthermore the admissible range for $m_{\beta\beta}$ was extended here and numerical results are more convenient in some cases and give more precise m_i, $i = 1, 2, 3$ values.

5. Conclusions and discussion

As may be seen by comparison the mass values (10) and the mass values presented in the Tabl.1 the evaluations of absolute neutrino masses are rather precise at small $m_{\beta\beta}$ values in despite of taking into account uncertainties for experimental data. The values obtained are in accordance with the results of the Refs.[9, 10] within the accuracy dependent on the graphics presented and data used, along with the results of the Ref.[19], where conditions are considered which cause $m_{\beta\beta}$ values larger than 10^{-3} eV. This value is critical, it will be seen from the reasoning below. As apparent from the Tabl.1 the IH spectrum can not be realized at $m_{\beta\beta} < 0.01eV$, this fact is previously found in Ref.[9].

One can see, when $m_{\beta\beta} < 10^{-3}eV$, the neutrino mass values practically are insensitive to $m_{\beta\beta}$ values. The values are $m_1 = 0.0045eV$, $m_2 = 0.0103eV$,

$m_3 = 0.0505eV$. The same values of m_i, $i = 1, 2, 3$, have been obtained in Ref.[21]. Due to this fact there is a possibility to take into account small but nonzero values of η_{13} for the NH mass spectrum. In this case the small values of $m_3 s_{13}^2$, that is less than $10^{-3}eV$, cannot change m_i values either. So we obtain the following condition $s_{13}^2 < 2 \cdot 10^{-2}$, under this condition and the previous condition: $m_{\beta\beta} < 10^{-3}eV$, the neutrino masses are equal to the invariable values.

The obtained estimations of neutrino masses at prescribed $m_{\beta\beta}$ values can be used for planning of experiments for a neutrinoless two beta decay search and for interpretation of results obtained. They can also be used for interpretation of any experimental results depended on absolute values of neutrino masses. For instance, the experiments in Troizk and Mainz for the measurement of a electron spectrum form in the β decay of tritium give the limit for a antineutrino mass $m(\bar{\nu}_e) < 2.2eV$ (95% C.L.) [22, 23], that is expected will be $0.2eV$ in the planned KATRIN experiment[24]. A statistical analysis of future neutrino mass experiments including neutrinoless double beta decay have been performed in Ref.[25] in order to check possibility of a reconstruction of a type of a neutrino mass spectrum.

Results of astrophysical and cosmological experiments lead to the following restriction on the sum of neutrino masses $\Sigma < 0.19$ [26]. It is significant that the domain of very small η_{13} and $m_{\beta\beta}$ values is practically accessible only for a theoretical examination. In the future we suppose to generalize the considered method for evaluation of neutrino masses, for instance, by taking into account effects of a CP noninvariance and sufficiently large η_{13} values, it makes possible to compare it with other methods for evaluation of absolute values of neutrino masses, in particular, with the three-flavor model for Majorana neutrinos with Pauli symmetry [5]. Note that allowing for a CP noninvariance can change the results obtained for Majorana neutrinos even with small η_{13} values.

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References

[1] Pontecorvo B.M., J.Ex.Th.Ph 33, 549 (1957), 34, 247 (1958), 53, 1717 (1967) (in Russian).

[2] Maki Z., M. Nakagawa, S. Sakata, Prog. Theor. Phys. 28, 870 (1962).

[3] Bilenky S.M., C. Giunti, W. Grimus, Prog. Part. Nucl. Phys. 43, 1 (1999).

[4] Kayser B., Plenary talk on Int. Conf. Neutrino-2008, New Zeland.

[5] Yu.V. Gaponov, DAN 423, No.5, 621 (2008); Dokl. Phys. 53, No.12, 624 (2008).

[6] Harrison P.F., D.H. Perkins, W.G. Scott, Phys. Lett. B530, 167 (2002).

[7] Smirnov A.Y., arXiv:hep-ph/0402264 (2004); Raidal M., Phys. Rev. Lett. 93, 161801 (2004); Minakata H., A.Y. Smirnov, Phys. Rev. D70, 073009 (2004).

[8] Yu.V. Gaponov, V.V. Khruschov, S.V. Semenov, Yad. Fiz. 71, 163 (2008); Phys. Atom. Nucl. 71, No.1, 162 (2008).

[9] Pascoli S., S.T. Petcov, Phys. Lett. B544, 239 (2002), *ibid* B580, 280 (2004); Pascoli S., S.T. Petcov, T. Schwetz, Nucl. Phys. B734, 24 (2006).

[10] Bilenky S.M., A. Faessler, F. Šimkovic, Phys. Rev. D70, 033003 (2004).

[11] Petcov S.T., New J. Phys. 6, 109 (2004)

[12] Merle A., W. Rodejohann, Phys. Rev. D73, 073012 (2006); W. Rodejohann, arXiv:0804.3925v3[hep-ph] (2004).

[13] Black D. et. al., Phys. Rev. D62, 073015 (2000); He X.G., A. Zee, Phys. Rev. D68, 037302 (2003); W. Rodejohann, Phys. Lett. B 579, 127 (2004); Nasri S., J. Schechter, S. Moussa, Phys. Rev. D71, 093005 (2005).

[14] Branco G.C. et al., Phys. Lett. B 562, 265 (2003); Z.Z. Xing, Phys. Rev. D69, 013006 (2004); B.C. Chauhan, J. Pulido, M. Picariello, Phys. Rev. D73, 053003 (2006).

[15] Avignone F.T.(III), S.R.Elliot and J.Engel, Rev. Mod. Phys. 80, 481 (2008).

[16] Zel'dovich Ya.B. and M.Yu. Khlopov, JETP Lett. 34, 141 (1981).

[17] Klapdor-Kleingrothaus H.V. et al., Nucl. Phys. Proc. Suppl. 100, 309 (2001).

[18] Aalseth C.E. et al., Phys. Atom. Nuclei 63, 1225 (2000).

[19] Pascoli S., S.T. Petcov, arXiv:0711.4993[hep-ph] (2007).

[20] Amsler C. et al. (PDG), Phys. Lett. B667, 1 (2008).

[21] Fritzsch H., arXiv:0902.2817[hep-ph] (2009).

[22] Lobashev V. et al., Nucl. Phys. A 719, 153c (2003).

[23] Kraus C. et al., Eur. Phys. J. C33, s01 (2004).

[24] Wolf J., arXiv:0810.3281[physics.ins-det] (2008).

[25] Maneschg W., A. Merle, W. Rodejohann, arXiv:0812.0479[hep-ph] (2008).

[26] Fogli G.L. et al., arXiv:0809.2936[hep-ph] (2008).